Rational-Harmonic Balancing Approach to Nonlinear Phenomena Governed by Pendulum-Like Differential Equations

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This paper presents a new approach for solving accurate approximate analytical solutions for nonlinear phenomena governed by pendulum-like differential equations. The new approach couples Taylor series expansion with rational harmonic balancing. An approximate rational solution depending on a small parameter is considered. After substituting the approximate solution into the governing differential equation, this equation is expanded in Taylor series of the parameter prior to harmonic balancing. The approach gives a cubic equation, which must be solved in order to obtain the value of the small parameter. A method for transforming this cubic equation into a linear equation is presented and discussed. Using this approach, accurate approximate analytical expressions for period and periodic solutions are obtained. We also compared the Fourier series expansions of the analytical approximate solution and the exact one. This allowed us to compare the coefficients for the different harmonic terms in these solutions. These analytical approximations may be of interest for those researchers working in nonlinear physical phenomena governed by pendulum-like differential equations in fields such as classical mechanics, vibrations, acoustics, electromagnetism, electronics, superconductivity, optics, gravitation, and others.

Key words: Nonlinear Oscillator; Rational Harmonic Balance Method; Approximate Solutions; Nonlinear Pendulum.